



## A Simple Algorithm to Compute Short-Dated CMM Forwards

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### Overview of the Algorithm

In the note, we describe the simple algorithm implemented in the spreadsheet to compute short-dated CMM forwards. The goal is to compute the fixed rate in a fixed versus floating forward rate agreement, where the floating rate is the CMM rate observed on the forward date (fixed and floating payments are made on a forward date). See Appendix A for details on deriving the CMM rate fixing from TBA mortgage prices. The basic idea behind the algorithm is to generate TBA mortgage prices across a distribution of swap rates based on their durations and convexities, and simply output the forward rate as the expected CMM rate on this distribution, while ensuring that the expected TBA mortgage prices on the distribution equal the market-observed forward TBA mortgage prices. The user inputs the trade date, the observation date, short-dated implied volatility and a table of TBA coupon data. As shown in the spreadsheet, the six-step algorithm is extremely simple and we hope this will establish transparency in short-dated CMM forward pricing.

### Inputs

- Trade Date/Pricing Date ( cell B4 ) - the date on which TBA and Volatility market data is observed.
- Observation Date ( cell B5 ) - the date on which the floating CMM rate is observed ( also the date on which fixed and floating payments are exchanged).
- Implied Volatility
  - Swaption Vol ( cell B7 ) – denotes the annualized basis point implied volatility for a blend of swaption tenors expiring on the observation date. The blend should roughly reflect the blend of swap rate tenors contributing to current-coupon rate moves ( with weights determined by the partial durations of the current coupon TBA mortgage security<sup>3</sup> ).
  - Mortgage/Swaption Vol Ratio ( cell B8 ) - Because we are concerned with hedging moves in mortgage prices/rates, we need to consider the implied

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<sup>3</sup> For example, partial durations might be 22% to 2y, 31% to 5y, 34% to 10y, and 13% to 30y. The same percentages would be applied to the implied volatilities for swaptions with these tails.

volatility on short-dated mortgage options. We scale our volatility input by the ratio of the market premium for current coupon TBA mortgage options versus the theoretical premium implied solely from TBA mortgage durations and associated swaption volatilities<sup>4</sup>.

- Vol ( cell B9 ) - The effective volatility ( henceforth denoted by  $\sigma$  ) used in the algorithm appears in cell B9.
- A table ( cells D4 – N9 ) consisting of the liquid forward TBA mortgage coupons along with their prices, drops, durations and convexities.
- For certain tail scenarios where the mortgage current coupon is beyond the range of user-specified liquid coupons, we extrapolate the CMM rate based on a user-specified view of how CMM rate moves as a function of the level of swap rates in far out-of-the money tail scenarios. The user input for this tail behavior of the CMM rate is specified in cell B11.

## Steps of the Algorithm

### 1. Interpolate the Forward TBA Price

Because CMM is defined as the mortgage rate derived from 30-day forward settling TBAs, the relevant TBA settle date for the trade is computed by taking the Observation Date and adding 30 calendar days (cell G15). For each coupon, we then linearly interpolate (in cells G18 – G21) the forward price for that settle date cells by applying the computed percentages in each BMA settlement month (cells B17 – E17). Heretofore, these forward prices will be denoted as  $P_{CFwd}$ .

### 2. Convexity Correct the Forward TBA Price

We convexity correct the forward TBA prices to account for the fact that the CMM index does not have the negative convexity of TBAs. Essentially, we want to know where the TBAs should trade if they had no negative convexity. This correction ensures that the expected (or scenario-weighted) TBA mortgage price equals the market-observed forward TBA mortgage price.<sup>5</sup>

The convexity correction is computed in cells C27– C30. The formula for the corrected price is as follows:

$$P_C = P_{CFwd} + \frac{1}{2} * Conv_C * \sigma^2 t$$

where  $P_{CFwd}$  is the forward TBA price from step 1,  $Conv_C$  is the user-input convexity from cells N6 – N9,  $\sigma$  is the volatility from cell B9, and  $t$  is the time in years from the trade date to the observation date. See Appendix B for the derivation of this formula.

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<sup>4</sup> There are fundamental as well as technical reasons for this. Questions can be addressed to the mortgage options desk.

<sup>5</sup> Because of numerical limitations of the spreadsheet, a very small basis can exist between the probability-weighted price and the forward price. The effect, computed in cells AI38 – AI41, is quite small.

### 3. Creating Distribution of Swap Rates

We create a discrete distribution of the level of swap rates ( denoted  $x$  ). It is convenient to represent the distribution in terms of shocks to the level of swap rates from the expected level ( shock amount  $\Delta x = x - E[x]$  ). We ensure that the variance of this discrete distribution is equal to the market-observed variance ( =  $\sigma^2 t$  where  $t$  is the time in years from the trade date to the observation date ). The discrete distribution is prepared in cells A33 – AF35.

### 4. Generate TBA Mortgage Prices across the Swap Rate Distribution

We project TBA mortgage prices as purely quadratic functions of the level of swap rates,

$$P_C(x) = P_C(E[x]) + Dur_C(E[x]) * \Delta x + 1/2 * Conv_C(E[x]) * (\Delta x)^2$$

where  $P_C(E[x])$  is the convexity-corrected forward-price from Step 2 and  $Dur_C(E[x])$ ,  $Conv_C(E[x])$  are the user-input duration and convexity of coupon  $C$ . The TBA Mortgage price generation is done in cells A38 – AF41.

### 5. Generate CMM Rates across the Swap Rate Distribution

For each shock amount, we convert the stack of TBA mortgage coupon prices to the CMM rate using the standard linear interpolation-based algorithm for deriving CMM rate fixings (see Appendix A). For the limited scenarios in which we cannot compute the current coupon from the available coupons, we apply our projected ratio for mortgage rate versus swap rate moves in tail scenarios (in cell B11) to the swap rate change from the neighboring node. The CMM rate generation is done in cells A45 – AF56.

### 6. Output: Forward CMM Rate

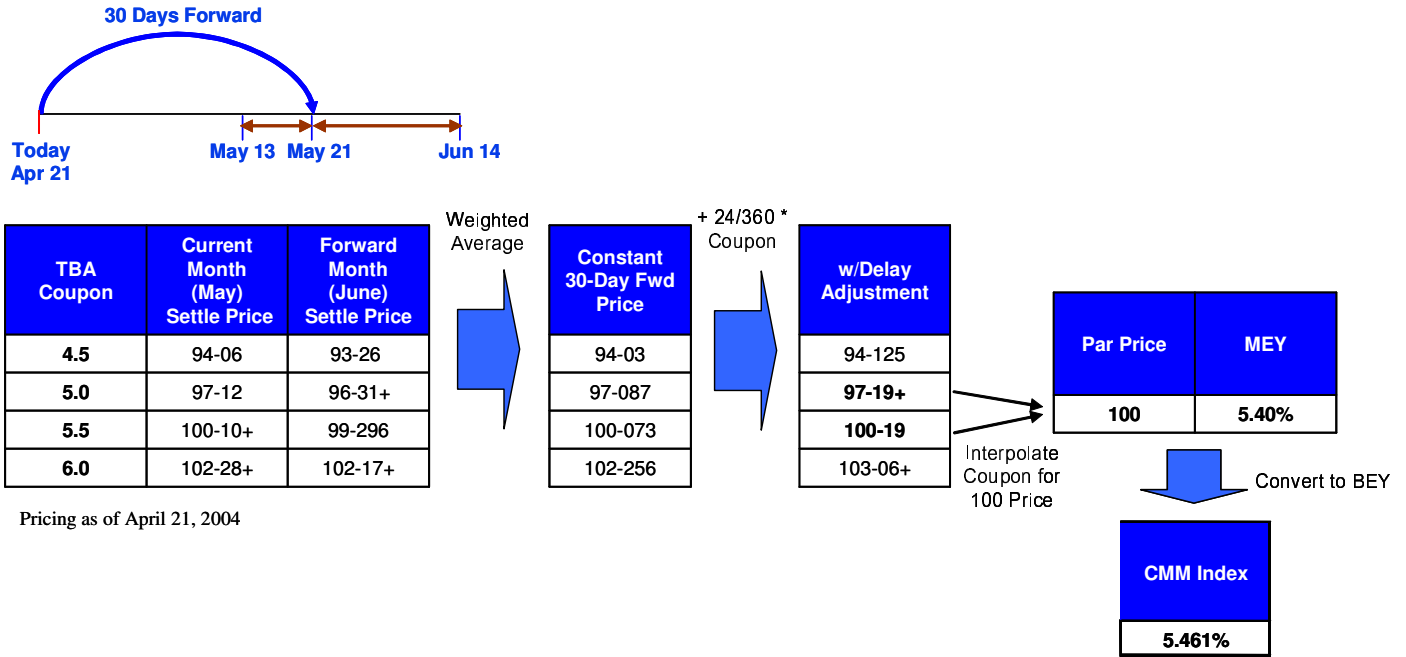
The forward CMM rate is computed as the expectation of the CMM rate random variable that was generated in the previous step. The sum-product of probabilities and CMM rates is done in cell B59.<sup>6</sup>

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<sup>6</sup> Forward price/rate is a weighted average price/rate with discounted risk-neutral probabilities as the weights. Since we are dealing with short-dated forwards, we can ignore the effect of variable discounting across the distribution and simply use risk-neutral probabilities as the weights.

# Appendix

## A. Algorithm to Calculate the CMM Rate Fiing



## B. Convexity Correction

Let  $Y(x)$  be a function of a random variable  $x$ . Then,

$$E[Y(x)] \approx E[Y(E[x]) + Y'(E[x]) * (x - E[x]) + \frac{1}{2} * Y''(E[x]) * (x - E[x])^2]$$

by Taylor series expansion around  $E[x]$  and neglecting third-order and higher terms.

Therefore,

$$E[Y(x)] \approx Y(E[x]) + \frac{1}{2} * Y''(E[x]) * Var[x]$$

We call  $Y''(E[x])$  as the convexity of  $Y$  and  $\frac{1}{2} * Y''(E[x]) * Var[x]$  as the associated convexity correction.

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